

University of Nebraska - Lincoln

DigitalCommons@University of Nebraska - Lincoln

---

Workshop Materials: Physics Teaching and the  
Development of Reasoning

ADAPT Program -- Accent on Developing  
Abstract Processes of Thought

---

1975

## Module 6: Analysis of Learning Materials

Francis P. Collea

*California State University, Fullerton*

Robert Fuller

rfuller@neb.rr.com

Robert Karplus

*University of California, Berkeley*

Lester G. Paldy

*SUNY, Stony Brook*

John W. Renner

*University of Oklahoma*

Follow this and additional works at: <https://digitalcommons.unl.edu/karplusworkshop>



Part of the [Science and Mathematics Education Commons](#)

---

Collea, Francis P.; Fuller, Robert; Karplus, Robert; Paldy, Lester G.; and Renner, John W., "Module 6: Analysis of Learning Materials" (1975). *Workshop Materials: Physics Teaching and the Development of Reasoning*. 7.

<https://digitalcommons.unl.edu/karplusworkshop/7>

This Article is brought to you for free and open access by the ADAPT Program -- Accent on Developing Abstract Processes of Thought at DigitalCommons@University of Nebraska - Lincoln. It has been accepted for inclusion in Workshop Materials: Physics Teaching and the Development of Reasoning by an authorized administrator of DigitalCommons@University of Nebraska - Lincoln.

## **Module 6 Analysis of Learning Materials**

### **Introduction**

Module 6 continues with the application of the concept of developmental stages in your physics teaching. The module concentrates on the analysis of physics texts and film loops, which provide important instructional inputs for students. As you read the excerpts we have selected for your review, keep in mind the characteristics of concrete and formal thought explained in Module 2. Also, remember that all students, regardless of their developmental stage, will find the text easier and will understand a new topic in a more broadly-based way if they can progress gradually from a concrete view of the subject. Of course, some students will progress further than others in grasping all the implications and subtleties contained in their reading.

### **Objectives**

To assist you in classifying text passages and film loops as to their requirements for concrete and formal reasoning patterns.

### **Procedure**

This module includes four text passages, two film loops and a review item to be analyzed for their demand on a student's reasoning patterns. We have highlighted certain features of these excerpts to indicate what makes a passage more or less accessible to the use of concrete reasoning only. In conclusion, we have listed criteria that you may use to evaluate physics texts or to help you prepare instructional materials of your own. Please choose a partner with whom you can work and exchange ideas during the module. Then use the activities in the attached instructional materials in the order given.

## Module 6 Instructional Materials

### 1. Excerpts A and B: Coulomb's Law

The first two excerpts we have chosen deal with Coulomb's law. Since the mathematical formulation of Coulomb's law makes use of direct and inverse proportions, formal reasoning is undoubtedly required for full comprehension. Nevertheless, a careful explanation that takes into account concrete thinking patterns can help the concrete or transitional students, present in substantial numbers in high school and college classes, grasp some of the underlying relationships among force, distance, and magnitude of charge, at least qualitatively. The formal thinker is also going to be helped to a richer understanding, achieved more easily, by such an explanation.

An important matter not identifiable from the excerpts is the student's concept of force. If force was defined in terms of actions and examples (deformation of a spring or rubber band, bending of a beam, weight), the student at the stage of concrete thought will have a chance to enlarge his understanding through the electrostatic application. If force was defined in terms of other concepts (mass and acceleration), no presentation of Coulomb's law will be understandable in terms of concrete reasoning patterns.

In the margins next to the text passages we have identified items that require identifiable patterns of reasoning on the part of the reader. In our opinion, Excerpt A makes an effort to communicate by means of concrete patterns of reasoning, but Excerpt B does not. Please read the two excerpts now, discuss their content and the marginal notes with your partner, and then continue on to the next excerpts.

Excerpt A**Electric Charge and Electric Force**

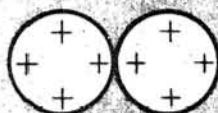
In an electrically neutral body the effects of positive and negative electric particles cancel. A positively charged body contains uncanceled positive particles, and a negatively charged body contains uncanceled negative particles. Thus the charge of a body depends on the uncanceled excess of positive or of negative particles, measured from neutral.

The force between two charged bodies depends on their separation and increases with the excess of positive or of negative electric particles on each body. Just how does the force depend upon the excess of electric particles? To answer this question we need a scheme to divide the excess of particles in a known way—in half, in thirds, etc. Suppose we touch a charged metal sphere with an identical uncharged sphere (Fig. 27-2). Then the electric particles will move around until they are shared equally by both spheres. Each sphere will have half the original charge.

(a)



(b)



(c)



The sharing of electric charge. When a charged sphere is touched to an identical uncharged one, the excess of electric particles divides equally. The final distribution of charge must be symmetrical, as shown in (c).

What happens to the electric forces when charges are shared? We measure the force of repulsion between two charged spheres *A* and *C* at a certain separation. Then we halve the charge on *A* by sharing it with an identical sphere *B*. The force of repulsion between *A* and *C* (still at the same separation) is *also* cut in half. Furthermore, we get the same force when *A* is replaced by *B*, the identical sphere with which it shared its charge. Apparently, charge and force are proportional, as we might have guessed.

Dependence on charge separation:

Explained in detail in the preceding section, which also illustrates the design and action of the torsion balance.

Charge sharing:

Reference to a sphere with eight positive charges by means of a diagram. Note, however, the unphysical conception suggested by the arrangement of charges in fig. 27-2b.

Comparison of Forces:

Identifies importance of keeping the same separation.

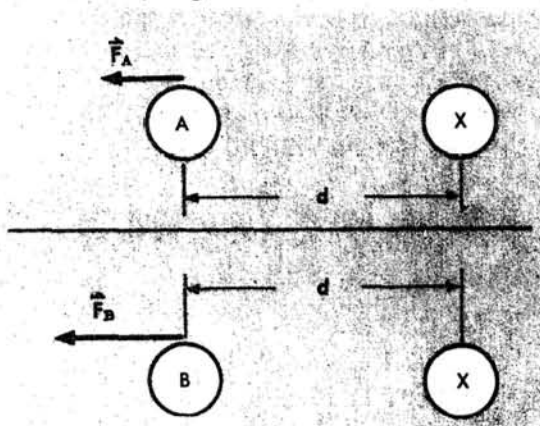
Such experiments give us a way of comparing charges quantitatively. Two charges are equal if they experience equal forces at a given distance from any third charge. One charge is twice another when it experiences twice the force. When a charge is halved by charge sharing, the force exerted on it by a third charge is also halved. In general, charges are compared by the ratio of the forces exerted on them by any other charge at a given distance. This *ratio* does not depend on the magnitude of the "other" charge nor on the distance apart (Fig. 27-3). Equivalently, we can compare the ratio of the forces exerted *on* the "other" charge by each of the two charges being compared.

Now let us summarize our knowledge in algebraic language. The electric force on a charge  $q$  is proportional to the charge:  $F \propto q$ . When this force is the force of interaction on the charge  $q$  by another small body of charge  $Q$ , the force is also proportional to the other charge. We can write this proportionality to both the charges as  $F \propto qQ$ .

We now have a definite meaning for charge, and we know how the electric force depends on the charges. We can combine this knowledge with Coulomb's experiments. They tell us that the force is inversely proportional to the square of the separation  $r$  between the charges. So we arrive at the complete expression for the force of interaction between two charges. The magnitude of the force on either charged body is

$$F = \frac{kQq}{r^2},$$

where the proportionality factor  $k$  depends only on the units in which we measure forces, separations, and charges.



To compare two charges, A and B, we place them in turn at the same distance from any other charge X, and measure the forces. The ratio of the charges equals the ratio of the forces:  $q_A/q_B = F_A/F_B$ . What do you think is the ratio of the forces exerted on X?

### Comparison of charges:

Interrupts the explanation and is therefore not correctly placed for a concrete-thinking reader who is concentrating on how the electric force depends on the magnitude of the charge.

### Applicability to point charges:

Bodies described as "small."

Excerpt B**COULOMB'S LAW**

The first quantitative investigation of the law of force between charged bodies was carried out by Charles Augustin de Coulomb (1736-1806) in 1784, utilizing for the measurement of forces a torsion balance of the type employed 13 years later by Cavendish in measuring gravitational forces. Coulomb found that the force of attraction or repulsion between two "point charges," that is, charged bodies whose dimensions are small compared with the distance  $r$  between them, is inversely proportional to the square of this distance.

The force also depends on the quantity of charge on each body. The net charge of a body might be described by a statement of the excess number of electrons or protons in the body. In practice, however, the charge of a body is expressed in terms of a unit much larger than the charge of an individual electron or proton. We shall use the letter  $q$  or  $Q$  to represent the charge of a body, postponing for the present the definition of the unit of charge.

In Coulomb's time, no unit of charge had been defined, nor had any method been developed for comparing a given charge with a unit. Despite this, Coulomb devised an ingenious method of showing how the force exerted on or by a charged body depended on its charge. He reasoned that if a charged spherical conductor were brought in contact with a second identical conductor, originally *uncharged*, the charge on the first would, by symmetry, be shared equally between the conductors. He thus had a method for obtaining one-half, one-quarter, and so on, of any given charge. The results of his experiments were consistent with the conclusion that the force between two point charges  $q$  and  $q'$  is proportional to the product of these charges. The complete expression for the force between two point charges is therefore

$$F = k \frac{qq'}{r^2}, \quad (24-1)$$

where  $k$  is a proportionality constant whose magnitude depends on the units in which  $F$ ,  $q$ ,  $q'$ , and  $r$  are expressed. Equation (24-1) is the mathematical statement of what is known today as *Coulomb's law*:

*The force of attraction or repulsion between two point charges is directly proportional to the product of the charges and inversely proportional to the square of the distance between them.*

Applicability to point charges:

"Point charges" identified as having small dimensions compared to the distance of separation, another ratio to be taken into account.

Dependence on charge separation:

Summarizes the result in one sentence, without reference to specific examples at this point.

Charge sharing:

Abstract reasoning and general conclusion only.

Comparison of forces:

Summarizes proportionality without reference to separation.



## 2. Excerpts C and D: Kinetic Energy

Study Excerpts C and D with your partner, taking note of the marginal comments as you did before. Evaluate their demand for concrete or formal patterns of reasoning, then compare with our evaluation on the next page.

### Excerpt C

#### 14-6 Kinetic energy

You probably have learned that the distance required to stop a car increases fourfold when its speed doubles. Have you ever wondered why? When a bicycle rider approaches a hill, he usually pedals as fast as he can so that he will get to the top of the hill more easily. Just how far up will his speed carry him? In both these examples, there is a transfer of energy from kinetic energy to another type: thermal energy of the brakes, or gravitational field energy of the bicycle, rider, and earth system.

As we have said in Chapter 4, kinetic energy is the energy stored in moving objects. Thus, the kinetic energy of the car determines how far it will advance as the brakes bring it to a stop. The bicyclist maximizes his kinetic energy as he approaches the hill.

When a force acts on a particle, its velocity or momentum changes, and usually its energy changes also. In this section we will derive a mathematical model for the relation of kinetic energy to speed. We will show how this relation can be used in conjunction with the law of conservation of energy to predict the motion of objects under many circumstances, such as the car coming to a stop and the bicycle moving uphill.

**Derivation.** Instead of constructing the model in the light of experimental results, we will derive it from Newton's theory. Imagine a particle at rest (zero speed, zero kinetic energy) that is acted upon by a constant net force until it is moving with the velocity  $v$ . The kinetic energy of the particle is, according to the law of conservation of energy, equal to the work done by the net force (Eq. 14-16). To find the work, we have to calculate the distance through which the particle moved while it was being accelerated by the action of the force.

This problem is very similar to the problem of free fall solved in Section 14-4. There, too, a constant force speeded up a particle that was initially at rest. The principal differences between that and the present task are that now the force can be any force (not only the force of gravity), and the motion can occur in any direction (not only vertically). Still, the motion and the force are in the same direction, because the particle starts from rest (Fig. 14-20).

The relative position of the particle is equal to one half of the velocity times the time (Eq. 14-17 from Eq. 14-10). The net force also can be related to the actual velocity (equal to the change of velocity) and to the elapsed time (Eq. 14-18 from Eq. 14-5). Since the force, the velocity, and the relative position are all in the same direction, the component of the displacement along the force direction is equal to the magnitude of the relative position (Eq. 14-19). When the formulas are combined to calculate the work and therefore the kinetic energy, we obtain a mathematical model (Eq. 14-20).

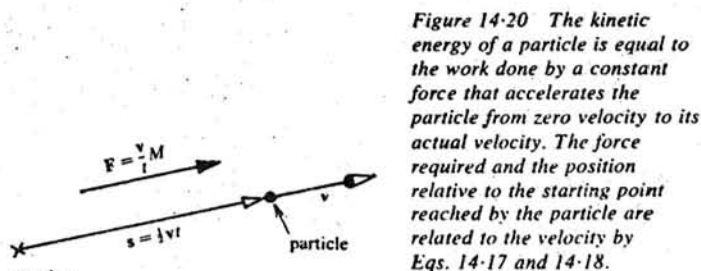


Figure 14-20 The kinetic energy of a particle is equal to the work done by a constant force that accelerates the particle from zero velocity to its actual velocity. The force required and the position reached by the particle are related to the velocity by Eqs. 14-17 and 14-18.

### Introduction:

Two familiar examples, described with reference to kinetic energy

### Feedback:

The concept of kinetic energy has evidently been introduced earlier, though not related to the mass and speed of the moving

### Introduction of a net force:

Anticipation of relation between kinetic energy and speed

### Selection of a constant force:

Reference to Newton's theory, but no rationale for constant force.

### Introduction of work:

Reference to energy conservation

### Algebraic derivation:

Leans on earlier treatment of free fall from rest which was illustrated with stroboscopic photographs and tables of time-distance data; the discussion paraphrases the equations very concisely and without new examples; keeps direction of motion general

### Final Conclusion:

The specific form of the speed-kinetic energy relation is exhibited

#### Equation 14-16

kinetic energy KE  
work W  
net force F  
displacement component along the force direction  $\Delta s_F$

$$KE = W = |F| \Delta s_F$$

#### Equation 14-17

position relative to starting point s  
velocity v  
elapsed time t

$$s = \frac{1}{2} vt$$

#### Equation 14-18

mass M

$$F = M \frac{v}{t}$$

#### Equation 14-19

speed v

$$\Delta s_F = |s| = \frac{1}{2} vt$$

#### Equation 14-20

$$KE = |F| \Delta s_F = M \frac{|v|}{t} \times \frac{1}{2} vt = \frac{1}{2} M v^2$$

Excerpt D**7-5 Kinetic Energy and the Work-Energy Theorem**

In our previous examples of work done by forces, we dealt with *unaccelerated* objects. In such cases the *resultant force* acting on the object is zero. Let us suppose now that the *resultant force* acting on an object is *not zero*, so that the object is *accelerated*. The conditions are the same in all respects to those that exist when a single unbalanced force acts on the object.

The simplest situation to consider is that of a *constant resultant force*  $F$ . Such a force, acting on a particle of mass  $m$ , will produce a constant acceleration  $a$ . Let us choose the  $x$ -axis to be in the common direction of  $F$  and  $a$ . What is the work done by this force on the particle in causing a displacement  $x$ ? We have (for constant acceleration) the relations

$$a = \frac{v - v_0}{t}$$

and

$$x = \frac{v + v_0}{2} \cdot t,$$

which are Eqs. 3-12 and 3-14 respectively (in which we have dropped the subscript  $x$ , for convenience, and chosen  $x_0 = 0$  in the last equation). Here  $v_0$  is the particle's speed at  $t = 0$  and  $v$  its speed at the time  $t$ . Then the work done is

$$\begin{aligned} W &= Fx = max \\ &= m \left( \frac{v - v_0}{t} \right) \left( \frac{v + v_0}{2} \right) t = \frac{1}{2}mv^2 - \frac{1}{2}mv_0^2. \end{aligned} \quad (7-11)$$

We call one-half the product of the mass of a body and the square of its speed the *kinetic energy* of the body. If we represent kinetic energy by the symbol  $K$ , then

$$K = \frac{1}{2}mv^2. \quad (7-12)$$

We may then state Eq. 7-11 in this way: *The work done by the resultant force acting on a particle is equal to the change in the kinetic energy of the particle.*

Introduction:

Completely abstract statement about forces, objects, and accelerations.

Feedback:

Reminder of behavior of unaccelerated objects to set the stage for now doing something else.

Introduction of a net force:

Reminder of relation between force and acceleration

Selection of a constant force:

Implied reference to Newton's law, but no rationale for constant force.

Introduction of work concept:

No rationale for suddenly asking about work

Algebraic derivation:

Quotes results from motion in one dimension with constant acceleration which was illustrated with time-distance and time-speed graphs; specializes to  $x$ -axis, but drops subscript.

Final conclusion:

The formula resulting from the algebraic operations is used to define the kinetic energy.

In our opinion, the first half of Excerpt C can be understood by the use of concrete patterns of reasoning and will therefore give all readers a better understanding (gut-feeling) of energy relationships. In spite of being intended for very different readers than Excerpt D, the remainder of Excerpt C is discouragingly similar to D. Still, the reader of C can omit the section entitled "Derivation" and come to grips with kinetic energy in a qualitative way; the reader of D gains at best a very formula-based notion of kinetic energy, with no idea how this "energy" is related to the energy he has met in his every-day life, chemistry courses, etc.



### 3. Communicating by means of Concrete Reasoning Patterns

By referring to the characteristics of concrete reasoning patterns described in Module 2, you can construct a list of items that will help you communicate at the concrete level. You can also review the features of formal reasoning patterns and then take special care to avoid these, or to call attention to those elements of formal thought that are used in the discussion because they appear unavoidable.

You can make a presentation more concrete by:

1. Beginning with concrete situations.
2. Illustrating the arguments with specific examples.
3. Providing "action models" or procedures that enable the student to work out an answer or verify a conclusion through concrete actions (in which he may often imagine himself) rather than through deductive or algebraic reasoning.
4. Providing a clear overview of a complicated explanation in advance, indicating the purpose and the principal steps.
5. Making clear references to formal operations when these are used:
  - a. Identify variables that are held fixed while others change;
  - b. State assumptions that are made;
  - c. Paraphrase equations in words, and don't use equations as principal parts of a sentence, " $E = mc^2$  is a consequence of Einstein's Relativity Theory;"
  - d. Use diagrams to illustrate steps of the reasoning;
  - e. Enumerate some specific instances when new classes or categories are defined.
6. Proceeding directly from known or previously explained ideas to new ones; don't start with "Let us assume that . . ." or "It is convenient to . . .".
7. Providing pictures of apparatus that is referred to.

### 4. Film Loops

After you and your partner have completed your work on the text passages, go to one of the film loop projection stations in the module area. You will find two film loops, (1) "Superposition of Pulses on a Spring" and (2) "Conservation of Energy." Please view them in the order 1-2, and read the film notes for each one so you can evaluate a student's reaction to them. Determine the loops' suitability in terms of their demand for concrete and formal reasoning patterns on the part of the viewer; use your experience with the text passages as basis for your analysis. Then read our comments on the next page. If you wish, view the loops a second time to examine their scenes more closely.

## 5. "Superposition of Pulses on a Spring"

This film loop would ordinarily be used by a student who had been introduced to the superposition principle in class discussion, or who had been asked to read about it in the text. The film does not attempt to provide a discussion of the principle, but only exhibits the phenomenon of superposition.

It will be apparent to you that this example can be quickly understood by a concrete operational thinker. The phenomenon of superposition is clearly shown at normal speed and in slow motion. Various aspects of the process that might be overlooked if only a single spring were used are highlighted by the ingenious technique of sending pulses along three identical springs, supported side by side. Note the way in which the demonstration is presented, proceeding from the simple to the more complex aspects of the phenomenon being illustrated. (For example, the longitudinal case followed the transverse illustration.)

Observe that even if a student overlooked the film notes, he would still derive a considerable amount of information from the film because of the direct way in which the phenomenon is presented. Since this film requires no formal reasoning operations to be performed by the viewer, it may be classified as suitable for concrete thinkers.

## 6. "Conservation of Energy"

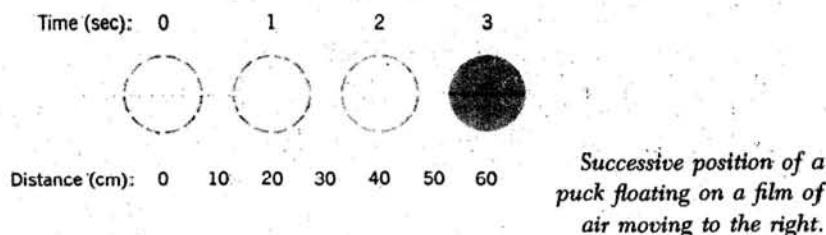
This film came to our attention when a graduate student who was using it with a group of freshmen in a physics course for non-science majors complained that the film did not contain enough information for the viewer to obtain the results quoted. The film shows a glider being accelerated along an air track. The captions assert that one can show from the data provided that the work done on the glider is equal to the change in the glider's kinetic energy. Can you identify the problem that the graduate student was having? How might you modify the film to make it at least partly understandable by the use of concrete reasoning patterns? (Hint: re-read our comments about text excerpts C and D.)

## Module 6 Review Questions

Excerpt E

Excerpt E is presented for your reading and analysis. At its conclusion on the next page we have posed four questions related to the sections numbered in the margin.

Several times we have used the phrase "uniform motion." Precisely what does this mean? Consider the motion of an air puck on a horizontal surface. Figure 6-2 illustrates such a puck moving to the right. The circles represent positions that the puck occupied at different times as it moved. These positions might have been determined by examining suc-



cessive frames of film taken by a motion-picture camera. As we see from Figure 6-2, the distance traveled by the puck in each 1.0-sec interval is the same, namely 20 cm. Assume, now, that the speed of the camera is doubled. The time between successive frames would be reduced to 0.5 sec. If the puck is engaged in uniform motion, then the distance between any two successive positions of the puck would be 10 cm. If, for *any* equal time intervals we choose the distance intervals are also equal, then the motion is uniform.

The *speed* of an object in uniform motion is defined as the ratio of a distance interval to the corresponding time interval. This can be written as an equation:

$$\text{speed} = \frac{\text{distance interval}}{\text{time interval}} \quad (6-1)$$

Usually scientists prefer to write such equations in symbols. The symbol commonly used for speed is  $v$ . The  $v$  really stands for velocity. To specify the velocity completely, you must know not only the speed but also the direction of the motion. Until the direction of motion assumes more importance in our discussion, we will use the two words interchangeably. The distance interval can be thought of as the difference between two readings of position,  $x$ , read from a meter stick at rest parallel to the path of the object, and the time interval can be thought of as the difference between two readings of time,  $t$ , read from a clock. The symbol  $\Delta$  before a quantity means a change in that quantity, so Equation 6-1 symbolically becomes

$$v = \frac{\Delta x}{\Delta t}$$

If the motion is not uniform, it is still possible to define an *average velocity*. Take any distance interval and divide by the corresponding time interval:

$$v_{av} = \frac{\Delta x}{\Delta t} \quad (6-3)$$

Equations 6-2 and 6-3 are very similar. The difference is that for uniform motion, the speed calculated from Equation 6-2 is independent of the interval selected, whereas for nonuniform motion, the average velocity calculated from Equation 6-3 may come out to be a large number for one particular interval and a small number for a different interval.

Please discuss your answers to these questions with your partner and/or other workshop participants. You might compare with the items on page 6-8 and the reasoning patterns described in Module 2. Our ideas are briefly described at the bottom of the page.

1. What reasoning pattern is required by the opening of the excerpt, Item 1? How might the opening have been made more concrete? How might it have been made more formal?
2. What level reasoning pattern is required to follow the generalization from the original example introduced in Item 2? Is this necessary to define uniform motion? Does it go far enough to define uniform motion? Could it have been done more clearly?
3. How does Item 3 help the reader? Should the text have given more emphasis to the directional requirements on uniform motion, possible in connection with Item 2? Should this explanation of the symbol  $v$  have been omitted?
4. In Item 4, the average velocity is defined by an arithmetic procedure. What level of reasoning pattern is involved in this definition? What level of reasoning pattern is involved in the explanation that relates this definition to the case of uniform motion? How well is the reader prepared for the transition from uniform to non-uniform motion?
5. Can you spot any sections that require formal reasoning patterns outside the numbered items? Explain your reasons and suggest other ways of handling the material. Do you have any comments on the overall organization of Excerpt E?

Brief answers: 1. Concrete in view of the example, but could have been improved by recalling one of the "several times."  
 2. Formal, especially when going to "any" time interval.  
 3. Seems unnecessary and confusing; the term "speed" could serve until direction is brought in.  
 4. Concrete for formula, formal for explanation; another puck example with unequal intervals would help.

### List of Sources

1. An Approach to Physical Science: Physical Science for Nonscience Students, (Brief Edition.) PSNS Project Staff and A. A. Strassenburg, John Wiley & Sons, Inc., N.Y. 1974 (Excerpt E)
2. Introductory Physics: A Model Approach, Robert Karplus, W. A. Benjamin, Inc., N.Y. 1969 (Excerpt C)
3. Physics, Part I, David Halliday and Robert Resnick, John Wiley & Sons, Inc., N.Y., 1966 (Excerpt D)
4. Physics, Second Edition. Physical Science Study Committee, D.C. Heath and Co., (Excerpt A).
5. College Physics, 4th Edition, Francis W. Sears, Mark W. Zemansky, Hugh D. Young, Addison-Wesley Publishing Co., Reading, Mass., 1974 (Excerpt B ).
6. "Superposition of Pulses on a Spring;" Loop No. 81293, Encyclopedia Britannica Educational Corporation (Super 8-Color).
7. "Conservation of Energy;" Loop No. 80-276, Ealing Corp., Cambridge, Mass. 02140.